

BLACK-SCHOLES AND BEYOND OPTION PRICING MODELS

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McGraw-Hill

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Library of Congress Cataloging-in-Publication Data

Chriss, Neil.

Black-Scholes and beyond : option pricing models / Neil Chriss.

p. cm.

Includes bibliographical references and index.

ISBN 0-7863-1025-1

1. Options (Finance)—Prices—Mathematical models. I. Title.

HG6024.A3C495 1997

332.64'5—dc20

96-17361

Printed in the United States of America

6 7 8 9 0 D O 3 2 1 0 9

by the delta of the option. The conclusion to all of this is:

The theoretical value of a vanilla European call option is completely determined by its delta, the risk-free rate of interest and the time to expiration.

Summary

Let's review the main points covered so far. First of all, we have shown that the Black-Scholes formula gives rise to a hedging strategy for the short position of a vanilla European call. To show that the value of the hedging portfolio is equal to that of the option at every time, we need to: 1) know the delta of the option at every time, and 2) use the delta to determine the correct value of B_t at every time t . There is only one thing left to do: give formulas for Δ_t and B_t .

4.8 THE BLACK-SCHOLES FORMULAS FOR Δ_t AND B_t

Now that we understand the general idea of the Black-Scholes formula, let's actually see what it is. All we have to do is to give the formulas for Δ_t and B_t .

The formulas are given in terms of the cumulative normal distribution function, discussed in Chapter 2. They are:

$$\Delta_t = N(d_1), \quad d_1 = \frac{\log(S_t/K) + (r + \frac{\sigma^2}{2})(T - t)}{\sigma \sqrt{T - t}} \quad (4.8.1)$$

$$B_t = N(d_2)K, \quad d_2 = \frac{\log(S_t/K) + (r - \frac{\sigma^2}{2})(T - t)}{\sigma \sqrt{T - t}} \quad (4.8.2)$$

where

S_t = price of stock per share at time t

K = strike price

r = risk-free rate of interest

σ = volatility of stock under geometric Brownian motion model

$T - t$ = time until expiration

$N(\cdot)$ = cumulative normal distribution function

Combining this with equation (4.6.1), we present the Black-Scholes formula for vanilla European call options on a non-dividend-paying stock:

$$C_t = N(d_1) \cdot S_t - e^{-r(T-t)} K \cdot N(d_2). \quad (4.8.3)$$